

V_{us} and precise Standard Model testsBARBARA SCIASCIA¹*INFN Laboratori Nazionali di Frascati, I-00044 Frascati, ITALY*

The recent significant progress on both the experimental and theoretical sides on the study of leptonic and semileptonic kaon decays allows to precisely test the Standard Model. Here we present results for the determination of $|V_{us}|$ from experimental data, the comparison between the values of $|V_{us}|$ obtained from data on $K \rightarrow \pi \ell \nu$ ($K_{\ell 3}$) and $K \rightarrow \mu \nu$ ($K_{\mu 2}$) decays, and tests of lepton universality in $K_{\ell 3}$ decays.

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A detailed analysis of precise tests of the Standard Model with leptonic and semileptonic kaon decays has already been presented in [1]. However, the recent significant progress on both the experimental and theoretical sides has motivated us to perform an updated analysis with three major areas of emphasis: the determination of $|V_{us}|$ from experimental data, with and without imposing CKM unitarity; the comparison between the values of $|V_{us}|$ obtained from data on $K \rightarrow \pi \ell \nu$ ($K_{\ell 3}$) and $K \rightarrow \mu \nu$ ($K_{\mu 2}$) decays and the corresponding constraints on deviations from the $V - A$ structure of the charged current; tests of lepton universality in $K_{\ell 3}$ decays. The complete work can be found in Ref [2]; here we report only on the main physics results.

Within the Standard Model (SM), leptonic and semileptonic kaon decays can be used to obtain the most accurate determination of the magnitude of the element V_{us} of the Cabibbo-Kobayashi-Maskawa (CKM) matrix. A detailed analysis of these processes potentially also provides stringent constraints on new physics scenarios: while within the SM, all $d^i \rightarrow u^j \ell \nu$ transitions are ruled by the same CKM coupling V_{ji} (satisfying the unitarity condition $\sum_k |V_{jk}|^2 = 1$), and G_F is the same coupling that governs muon decay, this is not necessarily true beyond the SM. New bounds on violations of CKM unitarity and lepton universality and deviations from the $V - A$ structure translate into significant constraints on various new-physics scenarios. Alternately, such tests may eventually turn up evidence of new physics. In the case of leptonic and semileptonic kaon decays, these tests are particularly significant given the large amount of data recently collected by several experiments, the substantial progress recently made in evaluating the corresponding hadronic matrix elements from lattice QCD, and the precise analytic calculations of radiative corrections and isospin-breaking effects recently performed within chiral perturbation theory, the low-energy effective theory of QCD.

An illustration of the importance of semileptonic kaon decays in testing the SM is provided by the unitarity relation $|V_{ud}|^2 + |V_{us}|^2 + |V_{ub}|^2 = 1 + \Delta_{\text{CKM}}$. Here the V_{ji} are the CKM elements as determined from the various $d^i \rightarrow u^j$ processes, where the value of G_F is determined from the muon life time: $G_\mu = 1.166371(6) \times 10^{-5} \text{GeV}^{-2}$. Δ_{CKM} parameterizes possible deviations from the SM induced by dimension-six operators, contributing either to muon decay or to $d^i \rightarrow u^j$ transitions [3]. The present accuracy on $|V_{us}|$ allows us to set bounds on Δ_{CKM} around 0.1%, which translate into bounds on the effective scale of new physics on the order of 10 TeV.

For each of the five decay modes for which rate measurements exist, we use

$$\Gamma_{K_{\ell 3}} = \frac{G_F^2 m_K^5}{192 \pi^3} C_K^2 S_{\text{EW}} \left(|V_{us}| f_+^{K^0 \pi^-}(0) \right)^2 I_{K\ell} \times (1 + \delta_{\text{EM}}^{K\ell} + \delta_{\text{SU}(2)}^{K\pi})^2$$

to evaluate $|V_{us}| f_+(0)$ from the decay rate data, the phase space integrals from dispersive fits, the long-distance radiative corrections, and the SU(2)-breaking corrections. We keep track of the correlations between the uncertainties on the values of $|V_{us}| f_+(0)$

Mode	$ V_{us} f_+(0)$	% err	BR	τ	Δ	Int	Correlation matrix (%)			
$K_L \rightarrow \pi e \nu$	0.2163(6)	0.26	0.09	0.20	0.11	0.06	+55	+10	+3	0
$K_L \rightarrow \pi \mu \nu$	0.2166(6)	0.29	0.15	0.18	0.11	0.08		+6	0	+4
$K_S \rightarrow \pi e \nu$	0.2155(13)	0.61	0.60	0.03	0.11	0.06			+1	0
$K^\pm \rightarrow \pi e \nu$	0.2160(11)	0.52	0.31	0.09	0.40	0.06				+73
$K^\pm \rightarrow \pi \mu \nu$	0.2158(14)	0.63	0.47	0.08	0.39	0.08				
Average	0.2163(5)									

Table 1: Values of $|V_{us}|f_+(0)$ as determined from each kaon decay mode, with approximate contributions to relative uncertainty (% err) from branching ratios (BR), lifetimes (τ), combined effect of $\delta_{\text{EM}}^{K\ell}$ and $\delta_{\text{SU}(2)}^{K\ell}$ (Δ), and phase space integrals (Int).

from different modes arising from the use of common corrections and from correlations in the input data set. The resulting values of $|V_{us}|f_+(0)$ are listed in Table 1.

To carry out this analysis, values are needed for the hadronic constant $f_+(0)$, as well as f_K/f_π (see in the following). These values are obtained using lattice QCD, and various determinations have been performed. The lattice QCD community, as represented by the FlaviaNet Lattice Averaging Group, is progressing towards convergence on a set of reference values, but in the meantime, for the purposes of this work, we are led to propose our own: $f_K/f_\pi=1.193(6)$ and $f_+(0)=0.959(5)$. Our adoption of these values is intended to illustrate what precision can be obtained in testing the SM, given current experimental and theoretical results. In particular, wherever possible we quote results for quantities such as $|V_{us}|f_+(0)$ or $|V_{us}/V_{ud}| \times f_K/f_\pi$, which are independent of lattice inputs and ready for use as new lattice results become available.

The principal experimental result of this review is the average value $|V_{us}|f_+(0)=0.2163(5)$, which has an uncertainty of about of 0.2%. The results from the five modes are in good agreement; the fit gives $\chi^2/\text{ndf} = 0.77/4$ ($P = 94\%$). Table 1 gives an approximate breakdown of the sources contributing to the total uncertainty on the determination of $|V_{us}|f_+(0)$ from each mode.

Equation

$$\frac{\Gamma_{K\ell 2}}{\Gamma_{\pi\ell 2}} = \frac{|V_{us}|^2 f_K^2 m_K (1 - m_\ell^2/m_K^2)^2}{|V_{ud}|^2 f_\pi^2 m_\pi (1 - m_\ell^2/m_\pi^2)^2} (1 + \delta_{\text{EM}})$$

, allows the ratio $|V_{us}/V_{ud}| \times f_K/f_\pi$ to be determined from experimental information on the radiation-inclusive $K_{\ell 2}$ and $\pi_{\ell 2}$ decay rates. The limiting uncertainty is that from $\text{BR}(K_{\mu 2(\gamma)})$, which is 0.28%. Using this, together with the value of $\tau_{K^\pm} = 12.384(15)$ ns and $\Gamma(\pi^\pm \rightarrow \mu^\pm \nu) = 38.408(7) \mu\text{s}^{-1}$, we obtain $|V_{us}/V_{ud}| \times f_K/f_\pi = 0.2758(5)$.

We determine $|V_{us}|$ and $|V_{ud}|$ from a fit to the results obtained above. As starting points, we use the value $|V_{us}|f_+(0) = 0.2163(5)$, together with the lat-

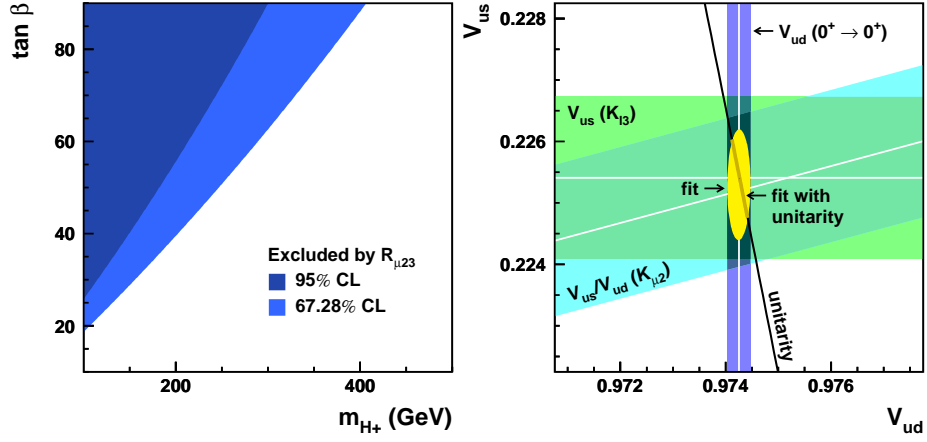


Figure 1: Left: Results of fits to $|V_{ud}|$, $|V_{us}|$, and $|V_{us}/V_{ud}|$. Right: Regions in the $(m_{H^\pm}, \tan \beta)$ plane in two-Higgs-doublet models excluded by the present result for $R_{\mu 23}$.

tice QCD estimate $f_+(0) = 0.959(5)$. We also use the result $|V_{us}/V_{ud}| \times f_K/f_\pi = 0.2758(5)$ together with the lattice estimate $f_K/f_\pi = 1.193(6)$. Thus we have $|V_{us}| = 0.2254(13)$ ($K_{\ell 3}$ only) and $|V_{us}/V_{ud}| = 0.2312(13)$ ($K_{\ell 2}$ only). Finally, we use the evaluation $|V_{ud}| = 0.97425(22)$ from a recent survey [4] of half-life, decay-energy, and BR measurements related to 20 superallowed $0^+ \rightarrow 0^+$ nuclear beta decays. Our fit to these inputs gives $|V_{ud}| = 0.97425(22)$, $|V_{us}| = 0.2253(9)$ ($K_{\ell 3}, K_{\ell 2}, 0^+ \rightarrow 0^+$), with $\chi^2/\text{ndf} = 0.014/1$ ($P = 91\%$) and negligible correlation between $|V_{ud}|$ and $|V_{us}|$. With the current world-average value, $|V_{ub}| = 0.00393(36)$, the first-row unitarity sum is then $\Delta_{\text{CKM}} = |V_{ud}|^2 + |V_{us}|^2 + |V_{ub}|^2 - 1 = -0.0001(6)$; the result is in striking agreement with the unitarity hypothesis. (Note that the contribution to the sum from $|V_{ub}|$ is essentially negligible.) As an alternate expression of this agreement, we may state a value for $G_{\text{CKM}} = G_\mu \sqrt{|V_{ud}|^2 + |V_{us}|^2 + |V_{ub}|^2}$. We obtain $G_{\text{CKM}} = 1.16633(35) \times 10^{-5} \text{ GeV}^{-2}$, with $G_\mu = 1.166371(6) \times 10^{-5} \text{ GeV}^{-2}$.

It is also possible to perform the fit with the unitarity constraint included, increasing by one the number of degrees of freedom. The constrained fit gives $|V_{us}| = \sin \theta_C = \lambda = 0.2254(6)$ (with unitarity) and $\chi^2/\text{ndf} = 0.024/2$ ($P = 99\%$). This result and that obtained above without assuming unitarity are both illustrated in Fig. 1. At this point, using $\Delta_{\text{CKM}} = 4 \left(\hat{\alpha}_{ll}^{(3)} - \hat{\alpha}_{lq}^{(3)} - \hat{\alpha}_{\phi l}^{(3)} + \hat{\alpha}_{\phi q}^{(3)} \right)$ [3] and the phenomenological value $\Delta_{\text{CKM}} = -0.0001(6)$, it is possible to set bounds on the effective scale of the four operators that parameterize new physics contributions to Δ_{CKM} . We obtain $\Lambda > 11 \text{ TeV}$ (90% C.L.). As noted in [3], for the operators $O_{ll}^{(3)}$, $O_{\phi l}^{(3)}$, and $O_{\phi q}^{(3)}$, this constraint is at the same level as the constraints from Z -pole measurements. For the four-fermion operator $O_{lq}^{(3)}$, Δ_{CKM} improves upon existing bounds from LEP2 by an order of magnitude.

An empirical value for the ratio

$$R_{\mu 23} = \left(\frac{f_K/f_\pi}{f_+(0)} \right)^{-1} \left(\left| \frac{V_{us}}{V_{ud}} \right| \frac{f_K}{f_\pi} \right)_{\mu 2} \frac{|V_{ud}|_{0^+ \rightarrow 0^+}}{[|V_{us}|f_+(0)]_{\ell 3}},$$

can be used to exclude regions of the $(m_{H^\pm}, \tan \beta)$ parameter space in models with two Higgs doublets, such as the minimal supersymmetry extension of the SM [5]. Operatively, we evaluate $R_{\mu 23}$ via a fit akin to that used to evaluate $|V_{us}|f_+(0)$, but with separate parameters accounting for the values of $|V_{us}|$ from $K_{\ell 3}$ and $K_{\mu 2}$ decays. The fit then has three free parameters: the value of $|V_{us}|$ from $K_{\ell 3}$ decays, the value of $|V_{us}/V_{ud}|$ from $K_{\mu 2}$ decays, and the value of $|V_{ud}|$ from $0^+ \rightarrow 0^+$ nuclear beta decays. The input values used for $|V_{us}|$ and $|V_{us}/V_{ud}|$ include the relevant lattice constants. The contribution to non-helicity-suppressed $K_{\ell 3}$ decays from charged Higgs exchange is negligible, so we include as a constraint in the fit the first-row unitarity condition on the value of $|V_{us}|$ from $K_{\ell 3}$ decays: $|V_{ud}|^2 + |V_{us}|_{K_{\ell 3}}^2 + |V_{ub}|^2 = 1$. Expressing the results of the fit in terms of $|V_{us}|$ from $K_{\ell 3}$ decays and the ratio $R_{\mu 23}$, we obtain $|V_{us}| = 0.2254(8)(K_{\ell 3}, 0^+ \rightarrow 0^+, \text{unitarity}), R_{\mu 23} = 0.999(7)(K_{\mu 2})$. The fit gives $\chi^2/\text{ndf} = 0.0003/1$ ($P = 99\%$), with $\rho = -0.55$ between the parameter uncertainties in the stated basis. The regions of the $(m_{H^\pm}, \tan \beta)$ parameter space in models with two Higgs doublets excluded at the 1σ and 95% CLs by this result for $R_{\mu 23}$ are shown as the shaded area in Fig. 1. Note that this result excludes the region at low m_{H^\pm} and large $\tan \beta$ favoured by $B \rightarrow \tau \nu$ [6].

As a general conclusion, we emphasize that the $\mathcal{O}(10 \text{ TeV})$ bound on the scale of new physics, which follows from the verification of the first-row CKM unitarity condition, represents one of the most stringent constraints on physics beyond the Standard Model.

References

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